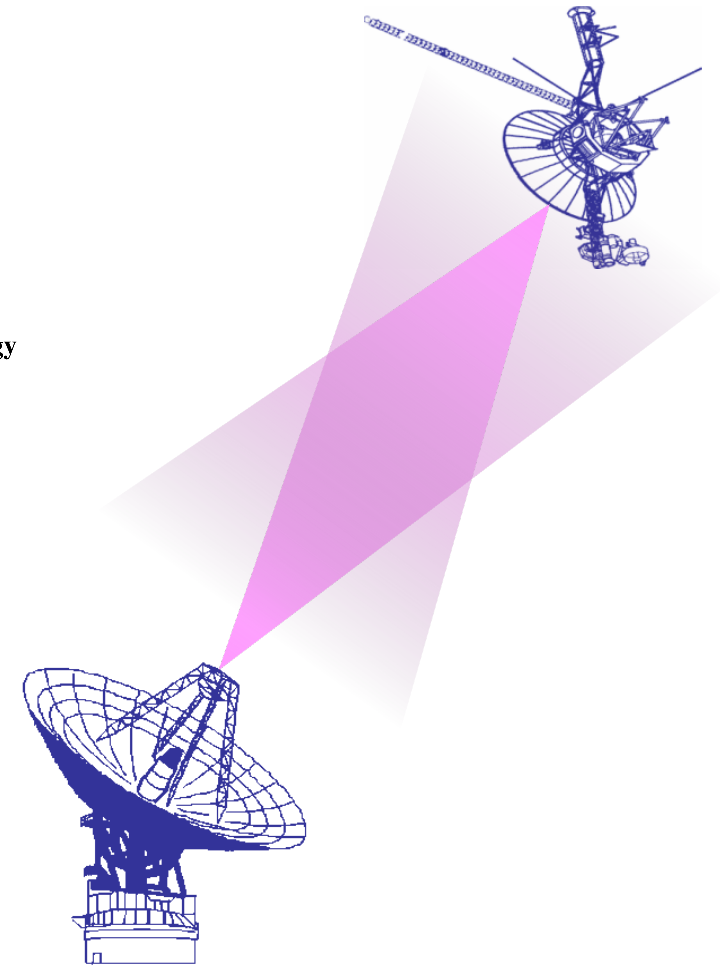
Telecommand/Telemetry Ranging for Deep-Space Applications



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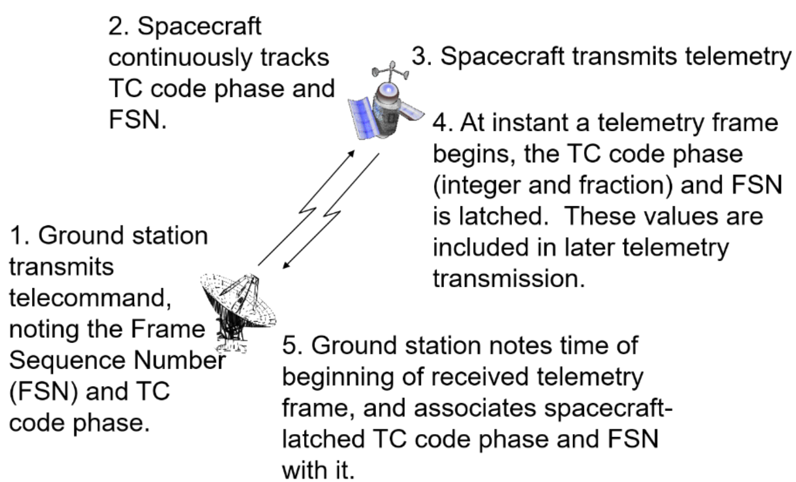
The uplink format:

The uplink is a sequence of Communications Link Transmission Units (CLTUs), possibly with idle between:

CLTU	CLTU	idle	CLTU	…

TC/TM ranging operates by tracking the number of symbols (integer and fractional) that have elapsed since the beginning of the most recently received CLTU. We call this symbol count the *TC code phase.*

Telecommand/telemetry ranging concept of operation:



Data flow:

1. A CLTU or idle sequence is sent on uplink, according to PLOP2 protocol; as it is transmitted, its TC code phase is periodically recorded and time-tagged on the ground, along with the FSN of the first TC frame within each CLTU.
2. The spacecraft continuously tracks the TC code phase of the acquired uplink CLTU/idle signal. The TC code phase shall be recorded as a 40 bit number, representing the number of symbols times 2^{20}, rounded to an integer, and stored in five octets, with bit 0 being the MSB and bit 39 being the LSB. At the moment when a telemetry frame with Frame Count FC is transmitted, the TC code phase shall be latched.
3. The (TC code phase, FC, FSN) triplet is transmitted to the ground as telemetry.
4. Range is computed from the known uplink frequency tuning history, ground TC code phase time-tag and FSN log, earth-receive time of telemetry frame FC, and (TC code phase, FC, FSN) triplet recorded by the spacecraft. The range computation algorithm remains nearly unchanged from telemetry ranging and PN ranging.

SIGNAL MODEL

The phase-modulated signal received at the spacecraft is modeled in complex exponential form:

$$s(t) = \mathrm{Re}\left(A\exp\{ j[\omega_0 t + \phi s(t-\tau) + \theta] \} \right) \qquad s(t-\tau) = \sum_i d_i p(t-\tau) \qquad d_i = \pm 1$$

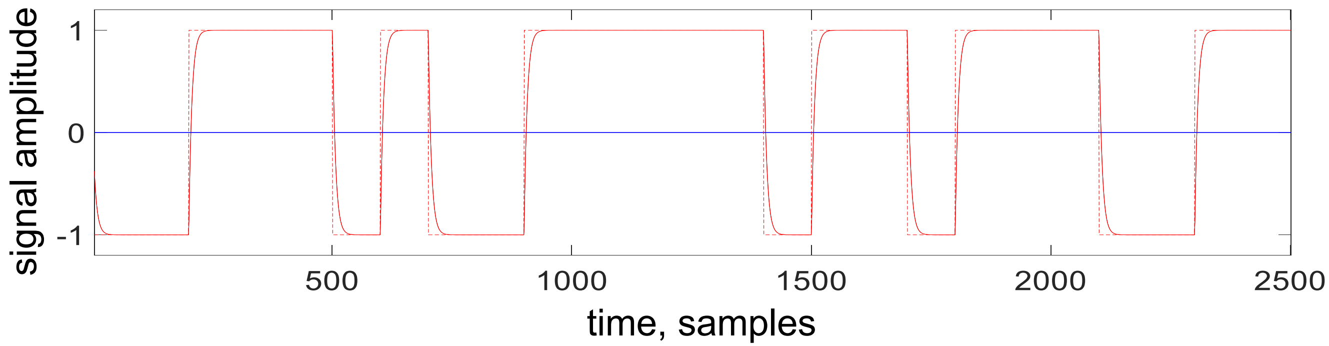


Figure 2. a) I-Q modulated unfiltered BPSK (dashed red), and filtered BPSK (solid red) Q component

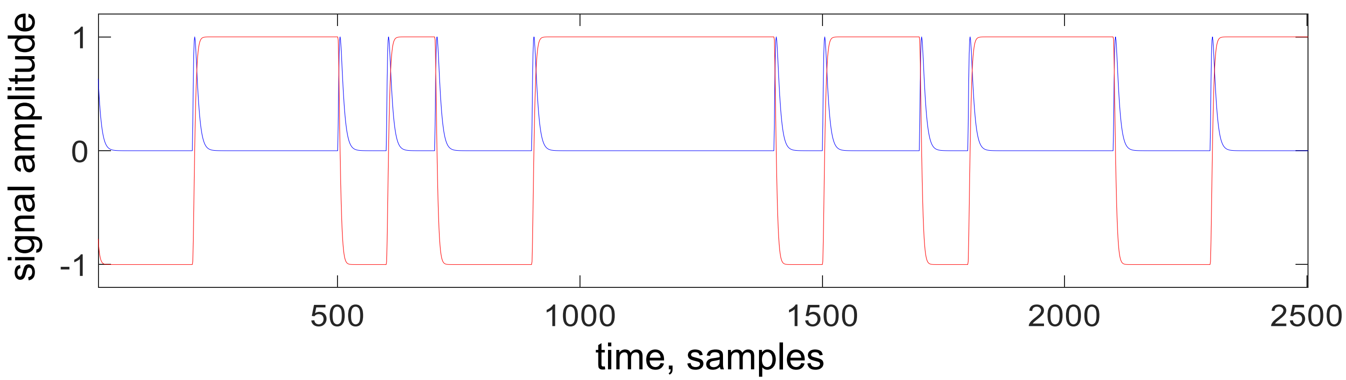


Figure 2. b) phase-modulated filtered BPSK: I (blue) and Q (red).

$$\tilde{r}_i = \tilde{s}_i(\tau) + \tilde{n}_i$$

Noise and received signal model: $\tilde{\mathbf{n}} = (\tilde{n}_0, \tilde{n}_1, \cdots, \tilde{n}_{N-1})$ $\quad \tilde{\mathbf{r}} = (\tilde{r}_0, \tilde{r}_1, \cdots, \tilde{r}_{N-1})$ $\quad \tilde{\mathbf{s}}(\tau) = [\tilde{s}_0(\tau), \tilde{s}_1(\tau), \cdots, \tilde{s}_{N-1}(\tau)]$

$$\sigma_n^2 = \sigma_{n,R}^2 + \sigma_{n,I}^2 \qquad \sigma_{n,R}^2 = \sigma_{n,I}^2$$

Noise and received signal statistics: $p(\tilde{\mathbf{n}}) = \left(\pi \sigma_n^2\right)^{-N} \prod_{i=0}^{N-1} \exp(-|\tilde{n}_i|^2 / \sigma_n^2)$, $\; p(\tilde{\mathbf{r}} \mid \tau) = \left(\pi \sigma_n^2\right)^{-N} \prod_{i=0}^{N-1} \exp(-|\tilde{r}_i - \tilde{s}_i|^2 / \sigma_n^2)$

Log-likelihood function: $\Lambda(\tilde{\mathbf{r}} \mid \tau) \equiv \ln[p(\tilde{\mathbf{r}} \mid \tau)]$ $\qquad \Lambda(\tilde{\mathbf{r}} \mid \tau) = -N \ln(\pi \sigma_n^2) - \frac{1}{\sigma_n^2} \sum_{i=0}^{N-1} |\tilde{r}_i - \tilde{s}_i(\tau)|^2$

Maximum likelihood (ML) estimate of delay, $\hat{\tau}$: that value of delay, τ, that maximizes $\Lambda(\tilde{\mathbf{r}} \mid \tau)$.

Expanding the square term and carrying out the maximization, yields:

$$\Lambda(\tilde{\mathbf{r}} \mid \tau) \equiv \ln[p(\tilde{\mathbf{r}} \mid \tau)] = -N \ln(\pi \sigma_n^2) - \frac{1}{\sigma_n^2} \sum_{i=0}^{N-1} |\tilde{r}_i - \tilde{s}_i(\tau)|^2 = -N \ln(\pi \sigma_n^2) - \frac{1}{\sigma_n^2} \sum_{i=0}^{N-1} \{|\tilde{r}_i|^2 + |\tilde{s}_i(\tau)|^2 - 2\,\mathrm{Re}[\tilde{r}_i\, \tilde{s}_i^*(\tau)]\}$$

Maximum Likelihood estimate of delay:

$$\boxed{\hat{\tau} = \max_{\tau} \Lambda(\tilde{\mathbf{r}} \mid \tau) = \max_{\tau} \sum_{i=0}^{N-1} \mathrm{Re}[\tilde{r}_i\, \tilde{s}_i^*(\tau)]}$$

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The Cramer-Rao lower bound (CRB) establishes the minimum error variance for any unbiased estimate

The CRB is based on the log-likelihood function. It can be expressed in two equivalent forms:

$$\mathrm{var}(\tau-\hat{\tau}) \geq \left(E\left\{ \left| \frac{\partial \Lambda(\mathbf{r}\,|\,\tau)}{\partial \tau} \right|^2 \right\} \right)^{-1}$$

preferred for delay estimation

$$\mathrm{var}(\tau-\hat{\tau}) \geq \left(E\left| \frac{\partial^2 \Lambda(\mathbf{r}\,|\,\tau)}{\partial \tau^2} \right| \right)^{-1}$$

leads to complicated math

Taking the first derivative of the log-likelihood function, making the substitution $\tilde{r}_i - \tilde{s}_i(\tau) = \tilde{n}_i$, squaring and taking the expectation of the random terms, yields:

$$\frac{\partial}{\partial \tau}\Lambda(\mathbf{r}\,|\,\tau) = -\frac{1}{\sigma_n^2}\sum_{i=0}^{N-1}\frac{\partial}{\partial \tau}|\tilde{r}_i - \tilde{s}_i(\tau)|^2 = \frac{2}{\sigma_n^2}\sum_{i=0}^{N-1}[\tilde{r}_i - \tilde{s}_i(\tau)]\frac{\partial \tilde{s}_i(\tau)}{\partial \tau} = \frac{2}{\sigma_n^2}\sum_{i=0}^{N-1}\tilde{n}_i\frac{\partial \tilde{s}_i(\tau)}{\partial \tau}$$

$$E\left\{ \left| \frac{\partial}{\partial \tau}\Lambda(\mathbf{r}\,|\,\tau) \right|^2 \right\} = \frac{4}{\sigma_n^4}E\left\{ \left| \sum_{i=0}^{N-1}\tilde{n}_i\frac{\partial \tilde{s}_i(\tau)}{\partial \tau} \right|^2 \right\} = \frac{4}{\sigma_n^4}\left\{ \sigma_n^2\sum_{i=0}^{N-1}\left| \frac{\partial \tilde{s}_i(\tau)}{\partial \tau} \right|^2 + \sum_{i=0}^{N-1}\sum_{\substack{j=0\\ j\neq i}}^{N-1}E(\tilde{n}_i\tilde{n}_j^*)\frac{\partial \tilde{s}_i(\tau)}{\partial \tau}\frac{\partial \tilde{s}_j^*(\tau)}{\partial \tau} \right\} = \frac{4}{\sigma_n^2}\sum_{i=0}^{N-1}\left| \frac{\partial \tilde{s}_i(\tau)}{\partial \tau} \right|^2$$

CRB for delay estimation:

$$\boxed{\mathrm{var}(\tau-\tau') \geq \left(E\left\{ \left| \frac{\partial}{\partial \tau}\Lambda(\mathbf{r}\,|\,\tau) \right|^2 \right\} \right)^{-1} = \frac{\sigma_n^2}{4}\left(\sum_{i=0}^{N-1}\left| \frac{\partial \tilde{s}_i(\tau)}{\partial \tau} \right|^2 \right)^{-1}}$$

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NUMERICAL RESULTS

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- Demod-remod process generates error-free replicas of *K* CLTUs that are used as reference
- Reference vector cross-correlated with stored received samples to obtain ML delay estimates
- Delay estimate transmitted to ground on the next available codeword, as described in [2,3]
- Performance of ML delay estimator for random data, with known demod-remod reference, is identical to that of ML estimator for PN codes

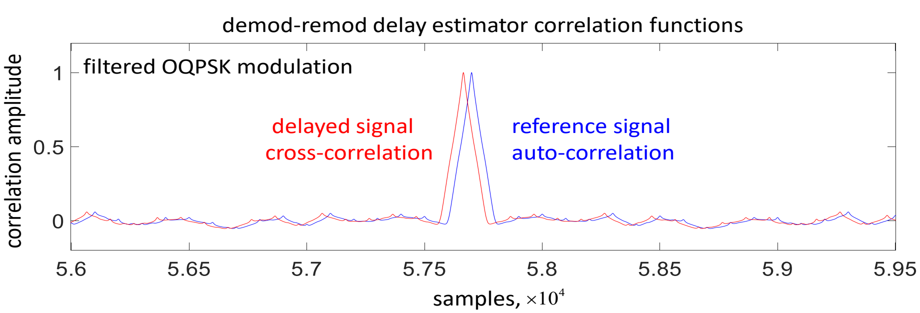

Figure 4. Filtered OQPSK auto-correlation function (blue) and cross-correlation function (red) at a sample-SNR of 15 dB; delay of 53 samples.

Delay estimator performance in high, medium, and low SNR regions

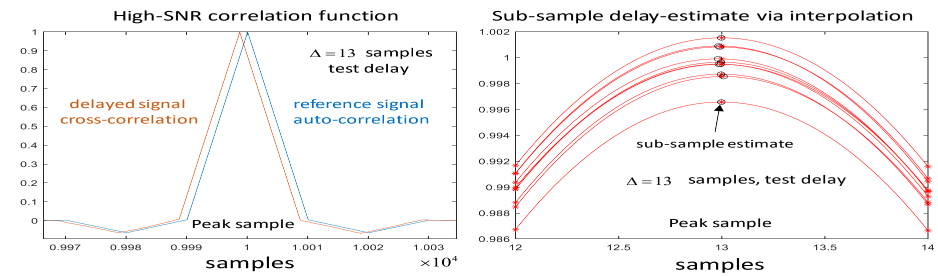

Figure 5. Correlation functions and fine delay estimates in the high-SNR regime

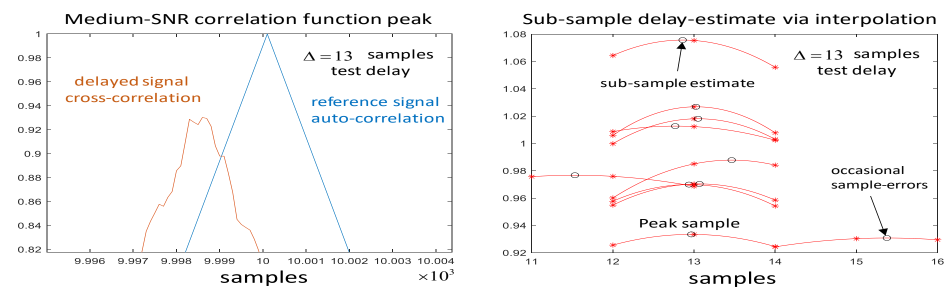

Figure 6. Correlation functions and fine delay estimates in the mid-SNR region

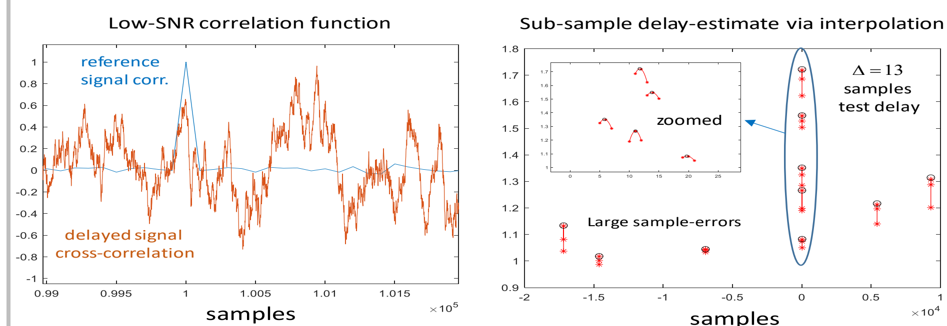

Figure 7. Correlation functions and fine delay estimates in the low-SNR region

NUMERICAL RESULTS - CONTINUED

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- ML delay estimator performance was determined for both BPSK and OQPSK signals
- ML approach was shown to achieve CRB at high SNR

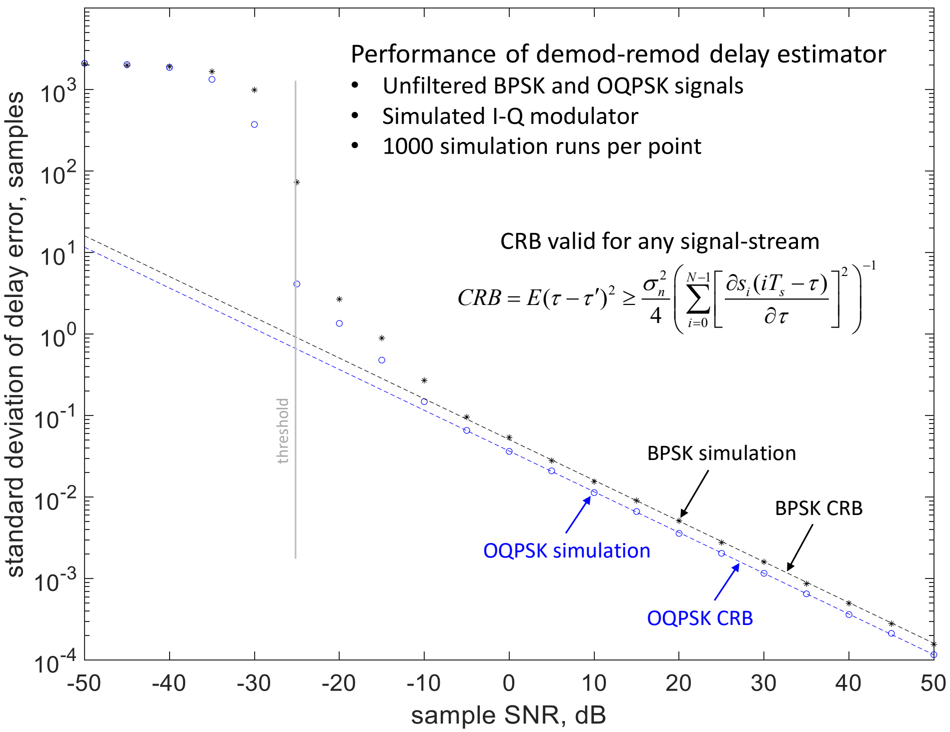

Figure 8. Comparison of ML delay estimator performance for unfiltered BPSK and OQPSK signals, with IQ modulation.

- Windowing at transitions shown to improve delay estimator performance
- Narrow uncertainty windows imply this approach best suited for low data-rate telemetry

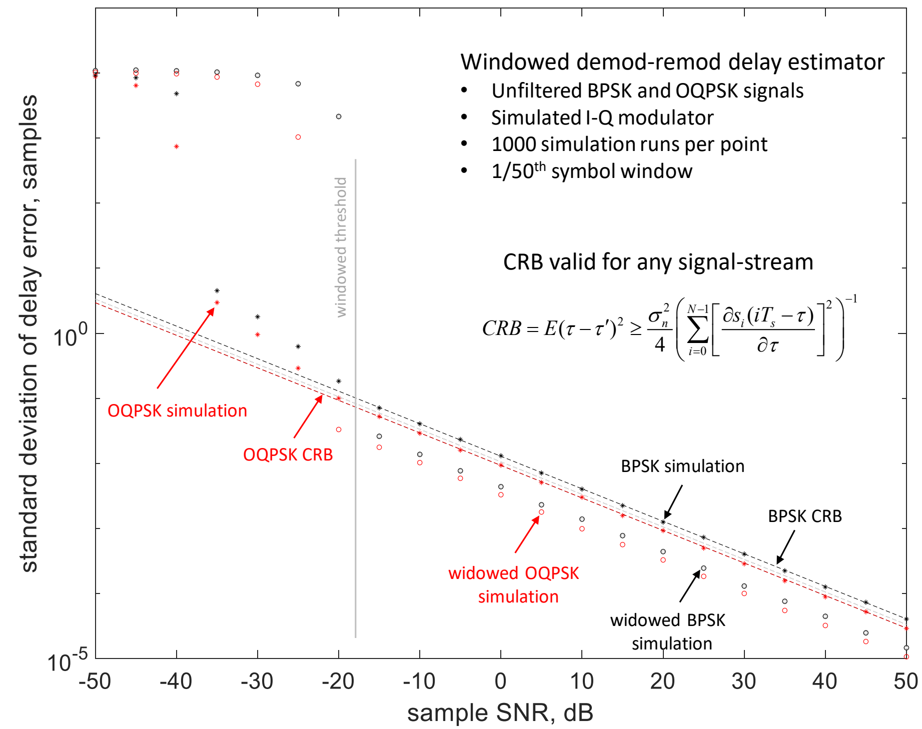

Figure 9. Improved delay estimator performance via windowing: unfiltered BPSK and OQPSK; I-Q modulation applied.

Theoretical performance of end-to-end ranging derived via Cramer-Rao bounds, and validated via MATLAB simulation

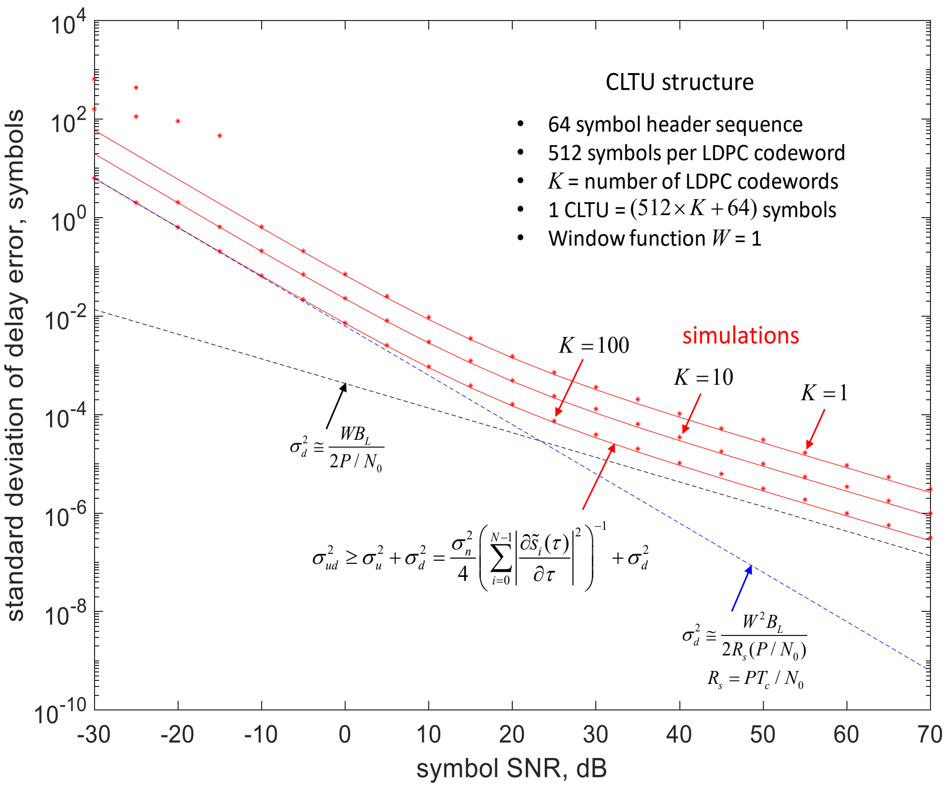

Figure 10. End-to-end rms delay error with OQPSK symbols as a function of symbol-SNR and using 100 samples/symbol, for $K = 1, 10$ and 100 LDPC codewords per CLTU: uplink CRB (dashed black); downlink DTTL low-SNR squaring loss bound (dashed blue); end-to-end performance bound (solid red); simulation (red asterisks)

Telemetry/Telecommand based ranging was shown to achieve same performance as PN-code based ranging

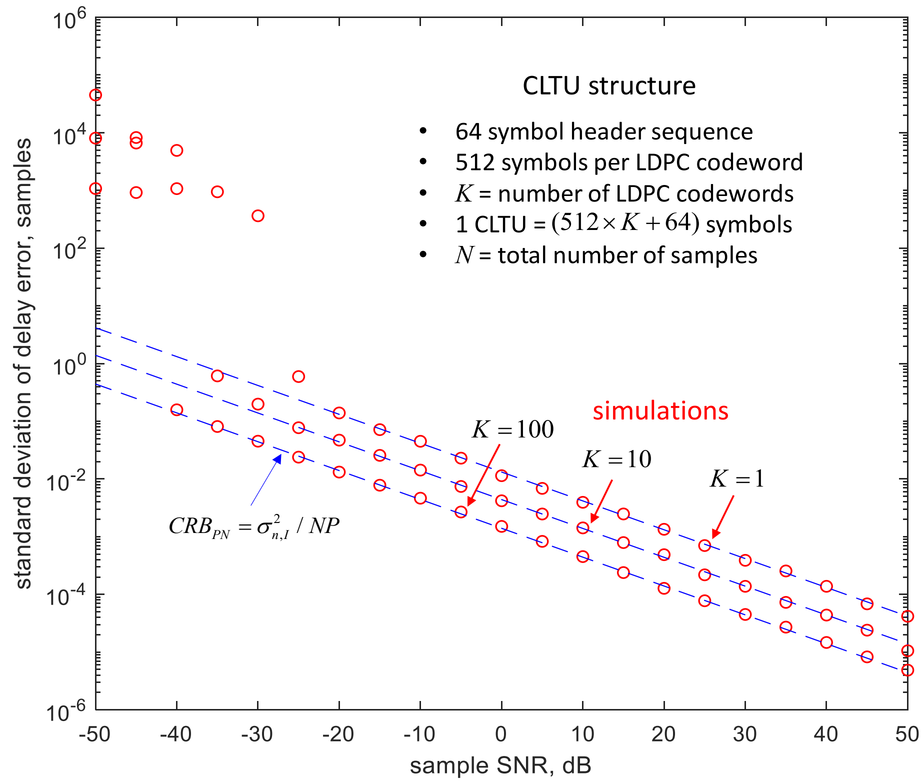

Figure 11. Comparison of demod-remod processing with random BPSK data (10 samples/symbol), and the high-SNR CRB for PN-coded chips in sample-SNR dB.

Summary and Conclusions

- Investigated delay and range estimation performance of two-way Deep-Space links
- Demod-remod process was utilized to generate error-free reference on future spacecraft
- Maximum likelihood algorithm for delay estimation with known reference was derived
- Cramer-Rao bounds on delay estimation error derived, compared to simulation results
- Developed pre-correlation windowing approach for up to 10 dB gain at low data-rates
- Less than 1m range error was demonstrated via analysis simulation, using realistic CLTUs with 100 or more LDPC codewords containing 512 symbols each